SL Paper 2

Given $f(x) = x^2 - 3x^{-1}, x \in \mathbb{R}, -5 \leqslant x \leqslant 5, x \neq 0,$

A football is kicked from a point A (a, 0), 0 < a < 10 on the ground towards a goal to the right of A.

The ball follows a path that can be modelled by **part** of the graph

$$y = -0.021x^2 + 1.245x - 6.01, x \in \mathbb{R}, y \geqslant 0.$$

x is the horizontal distance of the ball from the origin

y is the height above the ground

Both *x* and *y* are measured in metres.

i.a. Write down the equation of the vertical asymptote. [1]

i.b. Find $f'(x)$. [2]

i.c. Using your graphic display calculator or otherwise, write down the coordinates of any point where the graph of $y = f(x)$ has zero gradient. [2]

i.d. Write down all intervals in the given domain for which $f(x)$ is increasing. [3]

ii.a. Using your graphic display calculator or otherwise, find the value of *a*. [1]

ii.b. Find $\frac{dy}{dx}$. [2]

ii.c. (i) Use your answer to part (b) to calculate the horizontal distance the ball has travelled from A when its height is a maximum. [4]

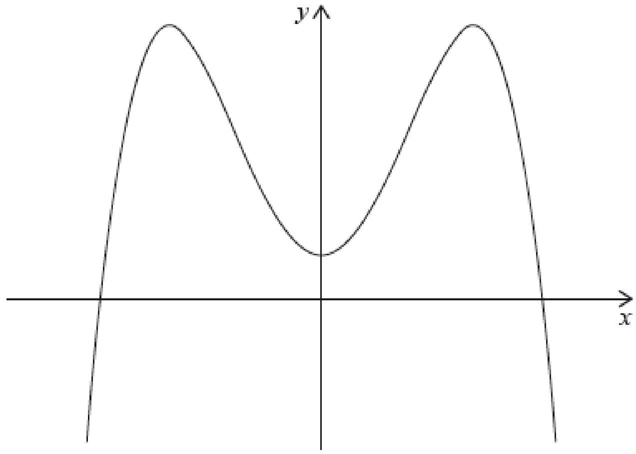
(ii) Find the maximum vertical height reached by the football.

ii.d. Draw a graph showing the path of the football from the point where it is kicked to the point where it hits the ground again. Use 1 cm to represent 5 m on the horizontal axis and 1 cm to represent 2 m on the vertical scale. [4]

ii.e. The goal posts are 35 m from **the point where the ball is kicked**. [2]

At what height does the ball pass over the goal posts?

Consider the function $f(x) = -x^4 + ax^2 + 5$, where a is a constant. Part of the graph of $y = f(x)$ is shown below.



It is known that at the point where $x = 2$ the tangent to the graph of $y = f(x)$ is horizontal.

There are two other points on the graph of $y = f(x)$ at which the tangent is horizontal.

a. Write down the y-intercept of the graph. [1]

b. Find $f'(x)$. [2]

c.i. Show that $a = 8$. [2]

c.ii. Find $f(2)$. [2]

d.i. Write down the x-coordinates of these two points; [2]

d.ii. Write down the intervals where the gradient of the graph of $y = f(x)$ is positive. [2]

e. Write down the range of $f(x)$. [2]

f. Write down the number of possible solutions to the equation $f(x) = 5$. [1]

g. The equation $f(x) = m$, where $m \in \mathbb{R}$, has four solutions. Find the possible values of m. [2]

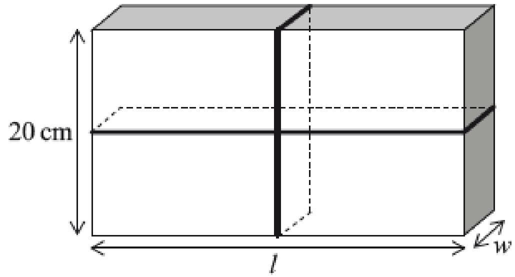
A parcel is in the shape of a rectangular prism, as shown in the diagram. It has a length l cm, width w cm and height of 20 cm.

The total volume of the parcel is $3000\ \text{cm}^3$.

a. Express the volume of the parcel in terms of l and w. [1]

b. Show that $l = \frac{150}{w}$. [2]

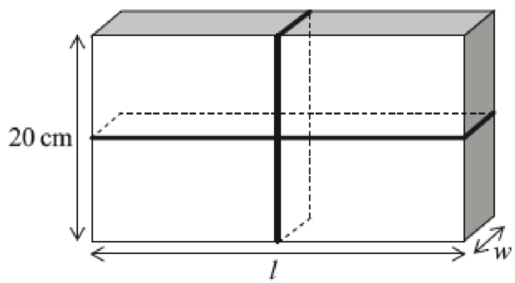
c. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. [2]



Show that the length of string, S cm, required to tie up the parcel can be written as

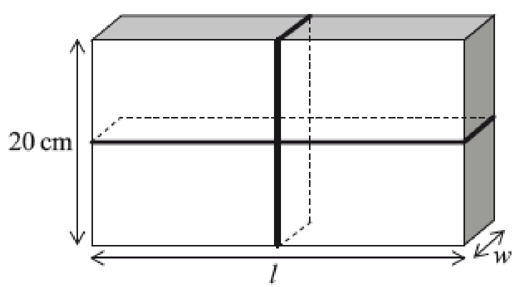
$$S = 40 + 4w + \frac{300}{w},\ 0 < w \leqslant 20.$$

d. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. [2]



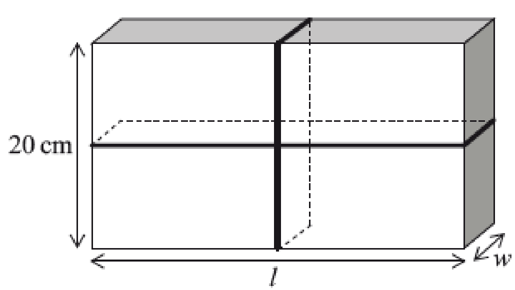
Draw the graph of S for $0 < w \leqslant 20$ and $0 < S \leqslant 500$, clearly showing the local minimum point. Use a scale of 2 cm to represent 5 units on the horizontal axis w (cm), and a scale of 2 cm to represent 100 units on the vertical axis S (cm).

e. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. [3]



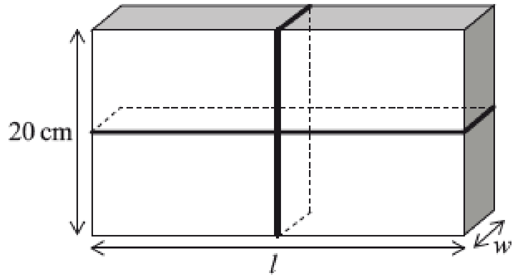
Find $\frac{\mathrm{d}S}{\mathrm{d}w}$.

f. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. [2]



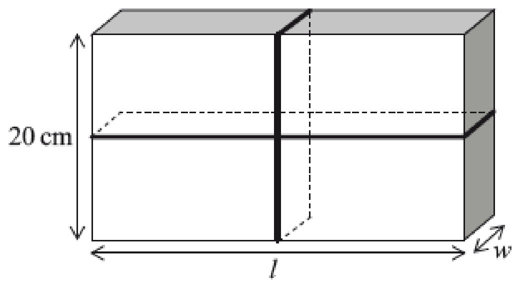
Find the value of w for which S is a minimum.

g. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. [1]



Write down the value, l, of the parcel for which the length of string is a minimum.

h. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. [2]



Find the minimum length of string required to tie up the parcel.

Consider the function $g(x) = x^3 + kx^2 - 15x + 5$.

The tangent to the graph of $y = g(x)$ at $x = 2$ is parallel to the line $y = 21x + 7$.

a. Find $g'(x)$. [3]

b.i. Show that $k = 6$. [2]

b.ii. Find the equation of the tangent to the graph of $y = g(x)$ at $x = 2$. Give your answer in the form $y = mx + c$. [3]

c. Use your answer to part (a) and the value of k, to find the x-coordinates of the stationary points of the graph of $y = g(x)$. [3]

d.i. Find $g'(-1)$. [2]

d.ii. Hence justify that g is decreasing at $x = -1$. [1]

e. Find the y-coordinate of the local minimum. [2]

Consider the function $f(x) = \frac{96}{x^2} + kx$, where k is a constant and $x \neq 0$.

a. Write down $f'(x)$. [3]

b. The graph of $y = f(x)$ has a local minimum point at $x = 4$. [2]

Show that $k = 3$.

c. The graph of $y = f(x)$ has a local minimum point at $x = 4$. [2]

Find $f(2)$.

d. The graph of $y = f(x)$ has a local minimum point at $x = 4$. [2]

Find $f'(2)$

e. The graph of $y = f(x)$ has a local minimum point at $x = 4$. [3]

Find the equation of the normal to the graph of $y = f(x)$ at the point where $x = 2$.

Give your answer in the form $ax + by + d = 0$ where $a,\ b,\ d \in \mathbb{Z}$.

f. The graph of $y = f(x)$ has a local minimum point at $x = 4$. [4]

Sketch the graph of $y = f(x)$, for $-5 \leqslant x \leqslant 10$ and $-10 \leqslant y \leqslant 100$.

g. The graph of $y = f(x)$ has a local minimum point at $x = 4$. [2]

Write down the coordinates of the point where the graph of $y = f(x)$ intersects the x-axis.

h. The graph of $y = f(x)$ has a local minimum point at $x = 4$. [2]

State the values of x for which $f(x)$ is decreasing.

A shipping container is to be made with six rectangular faces, as shown in the diagram.

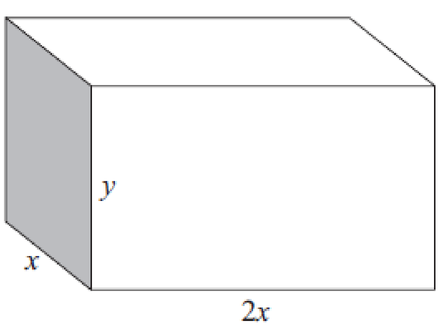


diagram not to scale

The dimensions of the container are

length $2x$
width x
height y.

All of the measurements are in metres. The total length of all twelve edges is 48 metres.

a. Show that $y = 12 - 3x$. [3]

b. Show that the volume V m^3 of the container is given by [2]

$V = 24x^2 - 6x^3$

c. Find $\frac{\mathrm{d}V}{\mathrm{d}x}$. [2]

d. Find the value of *x* for which *V* is a maximum. [3]

e. Find the maximum volume of the container. [2]

f. Find the length and height of the container for which the volume is a maximum. [3]

g. The shipping container is to be painted. One litre of paint covers an area of 15 m^2 . Paint comes in tins containing four litres. [4]

Calculate the number of tins required to paint the shipping container.

Consider the curve $y = x^3 + \frac{3}{2}x^2 - 6x - 2$.

a. (i) Write down the value of y when x is 2. [3]

(ii) Write down the coordinates of the point where the curve intercepts the y-axis.

b. Sketch the curve for $-4 \leqslant x \leqslant 3$ and $-10 \leqslant y \leqslant 10$. Indicate clearly the information found in (a). [4]

c. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [3]

d. Let L_1 be the tangent to the curve at $x = 2$. [8]

Let L_2 be a tangent to the curve, parallel to L_1.

(i) Show that the gradient of L_1 is 12.

(ii) Find the x-coordinate of the point at which L_2 and the curve meet.

(iii) Sketch and label L_1 and L_2 on the diagram drawn in (b).

e. It is known that $\frac{\mathrm{d}y}{\mathrm{d}x} > 0$ for $x < -2$ and $x > b$ where b is positive. [5]

(i) Using your graphic display calculator, or otherwise, find the value of b.

(ii) Describe the behaviour of the curve in the interval $-2 < x < b$.

(iii) Write down the equation of the tangent to the curve at $x = -2$.

Consider the function $f(x) = \frac{3}{4}x^4 - x^3 - 9x^2 + 20$.

a. Find $f(-2)$. [2]

b. Find $f'(x)$. [3]

c. The graph of the function $f(x)$ has a local minimum at the point where $x = -2$. [5]

Using your answer to part (b), show that there is a second local minimum at $x = 3$.

d. The graph of the function $f(x)$ has a local minimum at the point where $x = -2$. [4]

Sketch the graph of the function $f(x)$ for $-5 \leqslant x \leqslant 5$ and $-40 \leqslant y \leqslant 50$. Indicate on your sketch the coordinates of the y-intercept.

e. The graph of the function $f(x)$ has a local minimum at the point where $x = -2$. [2]

Write down the coordinates of the local maximum.

f. Let T be the tangent to the graph of the function $f(x)$ at the point $(2, -12)$. [2]

Find the gradient of T.

g. The line L passes through the point $(2, -12)$ and is perpendicular to T. [5]

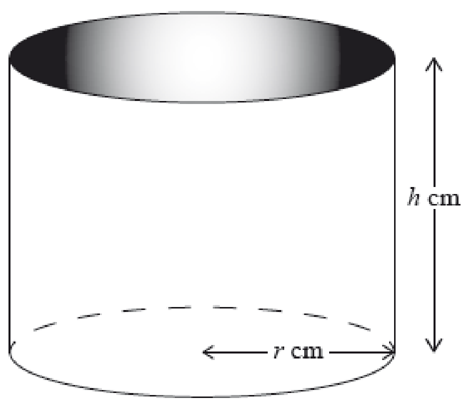
L has equation $x + by + c = 0$, where b and $c \in \mathbb{Z}$.

Find

(i) the gradient of L;

(ii) the value of b and the value of c.

A water container is made in the shape of a cylinder with internal height h cm and internal base radius r cm.



The water container has no top. The inner surfaces of the container are to be coated with a water-resistant material.

The volume of the water container is $0.5\ \mathrm{m}^3$.

The water container is designed so that the area to be coated is minimized.

One can of water-resistant material coats a surface area of $2000\ \mathrm{cm}^2$.

a. Write down a formula for A, the surface area to be coated. [2]

b. Express this volume in cm^3. [1]

c. Write down, in terms of r and h, an equation for the volume of this water container. [1]

d. Show that $A = \pi r^2 \frac{1\,000\,000}{r}$. [2]

d. Show that $A = \pi r^2 + \frac{1\,000\,000}{r}$. [2]

e. Find $\frac{\mathrm{d}A}{\mathrm{d}r}$. [3]

f. Using your answer to part (e), find the value of r which minimizes A. [3]

g. Find the value of this minimum area. [2]

h. Find the least number of cans of water-resistant material that will coat the area in part (g). [3]

Consider the function $f(x) = x^3 + \frac{48}{x}$, $x \neq 0$.

a. Calculate $f(2)$. [2]

b. Sketch the graph of the function $y = f(x)$ for $-5 \leqslant x \leqslant 5$ and $-200 \leqslant y \leqslant 200$. [4]

c. Find $f'(x)$. [3]

d. Find $f'(2)$. [2]

e. Write down the coordinates of the local maximum point on the graph of f . [2]

f. Find the range of f . [3]

g. Find the gradient of the tangent to the graph of f at $x = 1$. [2]

h. There is a second point on the graph of f at which the tangent is parallel to the tangent at $x = 1$. [2]

Find the x-coordinate of this point.

A, a. Sketch the graph of $y = 2^x$ for $-2 \leqslant x \leqslant 3$. Indicate clearly where the curve intersects the y-axis. [3]

A, b. Write down the equation of the asymptote of the graph of $y = 2^x$. [2]

A, c. On the same axes sketch the graph of $y = 3 + 2x - x^2$. Indicate clearly where this curve intersects the x and y axes. [3]

A, d. Using your graphic display calculator, solve the equation $3 + 2x - x^2 = 2^x$. [2]

A, e. Write down the maximum value of the function $f(x) = 3 + 2x - x^2$. [1]

A, f. Use Differential Calculus to verify that your answer to (e) is correct. [5]

B, a. The curve $y = px^2 + qx - 4$ passes through the point (2, –10). [2]

Use the above information to write down an equation in p and q.

B, b. The gradient of the curve $y = px^2 + qx - 4$ at the point (2, –10) is 1. [2]

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

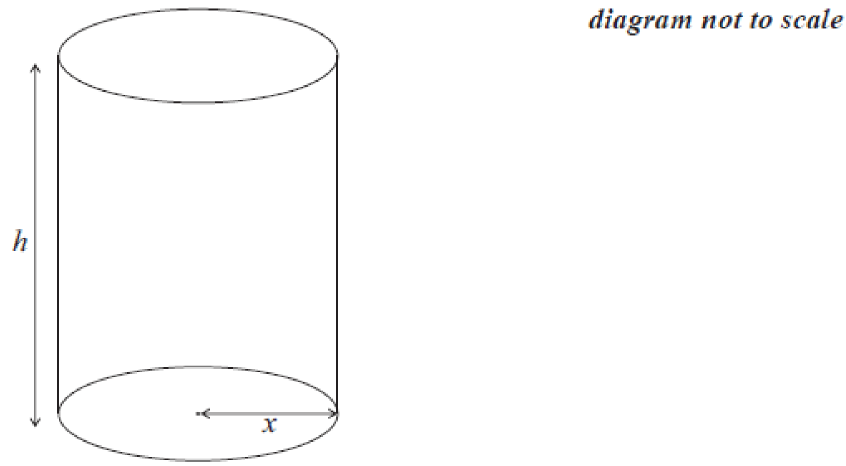
B, b, ii. The gradient of the curve $y = px^2 + qx - 4$ at the point (2, –10) is 1. [1]

Hence, find a second equation in *p* and *q*.

B, c. The gradient of the curve $y = px^2 + qx - 4$ at the point (2, –10) is 1. [3]

Solve the equations to find the value of *p* and of *q*.

A dog food manufacturer has to cut production costs. She wishes to use as little aluminium as possible in the construction of cylindrical cans. In the following diagram, *h* represents the height of the can in cm and *x*, the radius of the base of the can in cm.



The volume of the dog food cans is 600 cm^3.

a. Show that $h = \frac{600}{\pi x^2}$. [2]

b.i. Find an expression for the curved surface area of the can, in terms of *x*. Simplify your answer. [2]

b.ii. Hence write down an expression for *A*, the total surface area of the can, in terms of *x*. [2]

c. Differentiate *A* in terms of *x*. [3]

d. Find the value of *x* that makes *A* a minimum. [3]

e. Calculate the minimum total surface area of the dog food can. [2]

A function is defined by $f(x) = \frac{5}{x^2} + 3x + c,\ x \neq 0,\ c \in \mathbb{Z}$.

a. Write down an expression for $f'(x)$. [4]

b. Consider the graph of *f*. The graph of *f* passes through the point P(1, 4). [2]

Find the value of *c*.

c, i. There is a local minimum at the point Q. [4]

Find the coordinates of Q.

c, iiThere is a local minimum at the point Q. [3]

Find the set of values of *x* for which the function is decreasing.

d, iLet *T* be the tangent to the graph of *f* at P. [2]

Show that the gradient of *T* is –7.

d, iiLet *T* be the tangent to the graph of *f* at P. [2]

Find the equation of *T*.

e. *T* intersects the graph again at R. Use your graphic display calculator to find the coordinates of R. [2]

A function f is given by $f(x) = (2x + 2)(5 - x^2)$.

The graph of the function $g(x) = 5^x + 6x - 6$ intersects the graph of f.

a. Find the **exact** value of each of the zeros of f. [3]

b.i. Expand the expression for $f(x)$. [1]

b.ii. Find $f'(x)$. [3]

c. Use your answer to part (b)(ii) to find the values of x for which f is increasing. [3]

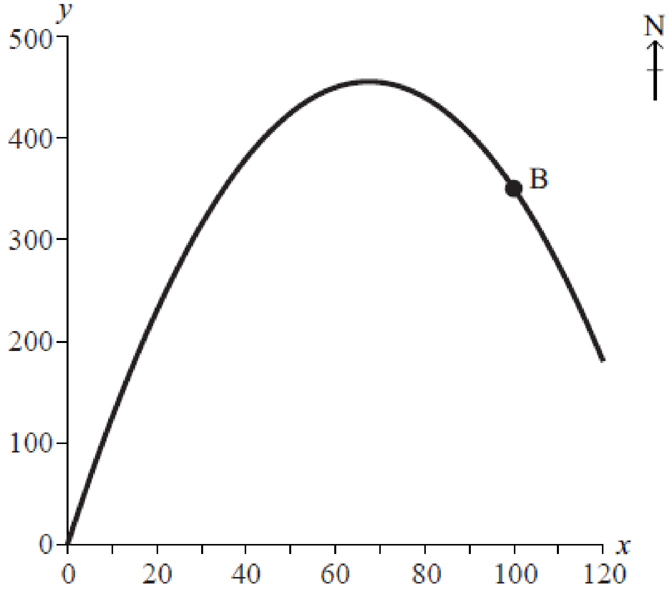
d. **Draw** the graph of f for $-3 \leqslant x \leqslant 3$ and $-40 \leqslant y \leqslant 20$. Use a scale of 2 cm to represent 1 unit on the x-axis and 1 cm to represent 5 units on the y-axis. [4]

e. Write down the coordinates of the point of intersection. [2]

The diagram shows an **aerial** view of a bicycle track. The track can be modelled by the quadratic function

$y = \frac{-x^2}{10} + \frac{27}{2}x$, where $x \geqslant 0,\ y \geqslant 0$

(x , y) are the coordinates of a point x metres east and y metres north of O , where O is the origin (0, 0) . B is a point on the bicycle track with coordinates (100, 350) .



a. The coordinates of point A are (75, 450). Determine whether point A is on the bicycle track. Give a reason for your answer. [3]

b. Find the derivative of $y = \frac{-x^2}{10} + \frac{27}{2}x$. [2]

c. Use the answer in part (b) to determine if A (75, 450) is the point furthest north on the track between O and B. Give a reason for your answer. [4]

d. (i) Write down the midpoint of the line segment OB. [3]

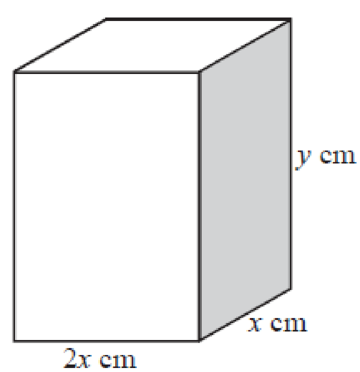
(ii) Find the gradient of the line segment OB.

e. Scott starts from a point C(0,150) . He hikes along a straight road towards the bicycle track, parallel to the line segment OB. [3]

Find the equation of Scott's road. Express your answer in the form $ax + by = c$, where a, b and $c \in \mathbb{R}$.

f. Use your graphic display calculator to find the coordinates of the point where Scott first crosses the bicycle track. [2]

A closed rectangular box has a height y cm and width x cm. Its length is twice its width. It has a fixed outer surface area of 300 cm^2 .



i.a. Factorise $3x^2 + 13x - 10$. [2]

i.b. Solve the equation $3x^2 + 13x - 10 = 0$. [2]

i.c. Consider a function $f(x) = 3x^2 + 13x - 10$. [2]

Find the equation of the axis of symmetry on the graph of this function.

i.d. Consider a function $f(x) = 3x^2 + 13x - 10$. [2]

Calculate the minimum value of this function.

ii.a. Show that $4x^2 + 6xy = 300$. [2]

ii.b. Find an expression for y in terms of x. [2]

ii.c. Hence show that the volume V of the box is given by $V = 100x - \frac{4}{3}x^3$. [2]

ii.d. Find $\frac{\mathrm{d}V}{\mathrm{d}x}$. [2]

ii.e. (i) Hence find the value of x and of y required to make the volume of the box a maximum. [5]

(ii) Calculate the maximum volume.

A lobster trap is made in the shape of half a cylinder. It is constructed from a steel frame with netting pulled tightly around it. The steel frame consists of a rectangular base, two semicircular ends and two further support rods, as shown in the following diagram.

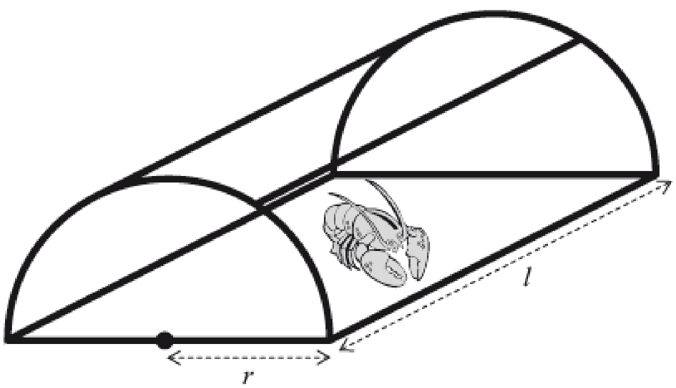


diagram not to scale

The semicircular ends each have radius r and the support rods each have length l.
Let T be the total length of steel used in the frame of the lobster trap.

a. Write down an expression for T in terms of r, l and π. [3]

b. The volume of the lobster trap is $0.75\ \mathrm{m}^3$. [3]

Write down an equation for the volume of the lobster trap in terms of r, l and π.

c. The volume of the lobster trap is $0.75\ \mathrm{m}^3$. [2]

Show that $T = (2\pi + 4)r + \frac{6}{\pi r^2}$.

d. The volume of the lobster trap is $0.75\ \mathrm{m}^3$. [3]

Find $\frac{\mathrm{d}T}{\mathrm{d}r}$.

e. The lobster trap is designed so that the length of steel used in its frame is a minimum. [2]

Show that the value of r for which T is a minimum is $0.719\ \mathrm{m}$, correct to three significant figures.

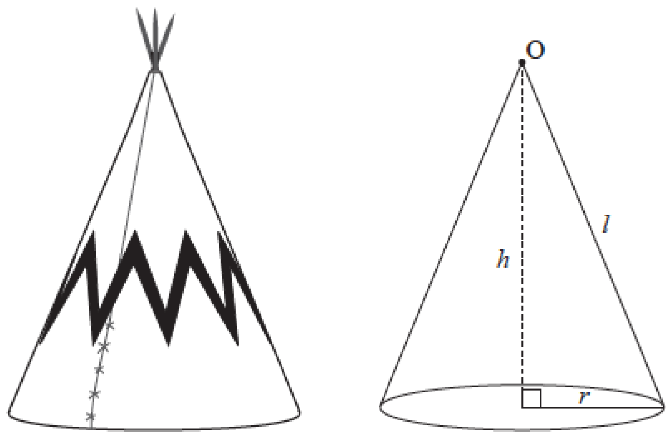
f. The lobster trap is designed so that the length of steel used in its frame is a minimum. [2]

Calculate the value of l for which T is a minimum.

g. The lobster trap is designed so that the length of steel used in its frame is a minimum. [2]

Calculate the minimum value of T.

Tepees were traditionally used by nomadic tribes who lived on the Great Plains of North America. They are cone-shaped dwellings and can be modelled as a cone, with vertex O, shown below. The cone has radius, r, height, h, and slant height, l.



A model tepee is displayed at a Great Plains exhibition. The curved surface area of this tepee is covered by a piece of canvas that is 39.27 m^2, and has the shape of a semicircle, as shown in the following diagram.

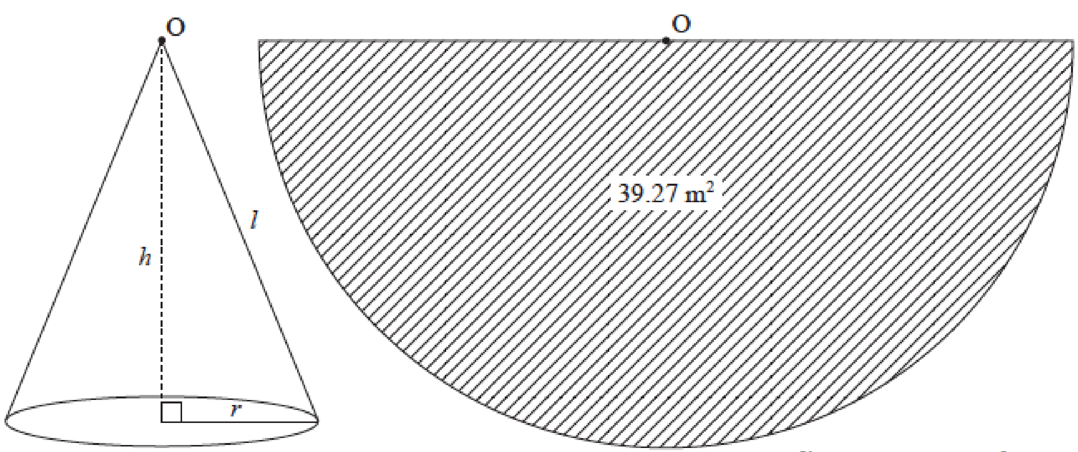


diagram not to scale

a. Show that the slant height, l, is 5 m, correct to the nearest metre. [2]

b. (i) Find the circumference of the base of the cone. [6]

(ii) Find the radius, r, of the base.

(iii) Find the height, h.

c. A company designs cone-shaped tents to resemble the traditional tepees. [1]

These cone-shaped tents come in a range of sizes such that the sum of the diameter and the height is equal to **9.33 m**.

Write down an expression for the height, h, in terms of the radius, r, of these cone-shaped tents.

d. A company designs cone-shaped tents to resemble the traditional tepees. [1]

These cone-shaped tents come in a range of sizes such that the sum of the diameter and the height is equal to **9.33 m**.

Show that the volume of the tent, V, can be written as

$$V = 3.11\pi r^2 - \frac{2}{3}\pi r^3.$$

e. A company designs cone-shaped tents to resemble the traditional tepees. [2]

These cone-shaped tents come in a range of sizes such that the sum of the diameter and the height is equal to **9.33 m**.

Find $\frac{\mathrm{d}V}{\mathrm{d}r}$.

f. A company designs cone-shaped tents to resemble the traditional tepees. [4]

These cone-shaped tents come in a range of sizes such that the sum of the diameter and the height is equal to **9.33 m**.

(i) Determine the exact value of r for which the volume is a maximum.

(ii) Find the maximum volume.

The function $f(x)$ is defined by $f(x) = 1.5x + 4 + \frac{6}{x}, x \neq 0$.

a. Write down the equation of the vertical asymptote. [2]

b. Find $f'(x)$. [3]

c. Find the gradient of the graph of the function at $x = -1$. [2]

d. Using your answer to part (c), decide whether the function $f(x)$ is increasing or decreasing at $x = -1$. Justify your answer. [2]

e. Sketch the graph of $f(x)$ for $-10 \leqslant x \leqslant 10$ and $-20 \leqslant y \leqslant 20$. [4]

f. P_1 is the local maximum point and P_2 is the local minimum point on the graph of $f(x)$. [4]

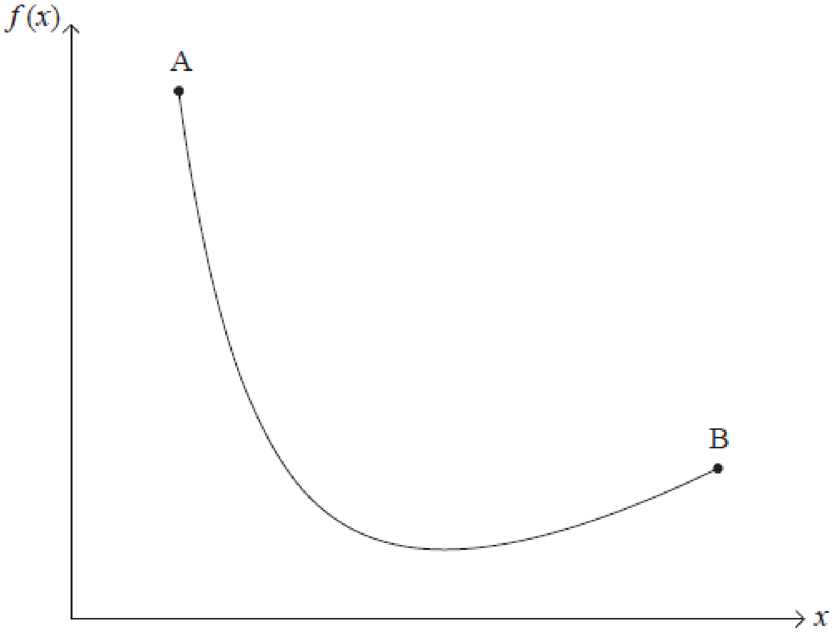
Using your graphic display calculator, write down the coordinates of

(i) P_1 ;

(ii) P_2 .

g. Using your sketch from (e), determine the range of the function $f(x)$ for $-10 \leqslant x \leqslant 10$. [3]

The graph of the function $f(x) = \frac{14}{x} + x - 6$, for $1 \leq x \leq 7$ is given below.



a. Calculate $f(1)$. [2]

b. Find $f'(x)$. [3]

c. **Use your answer to part (b)** to show that the *x*-coordinate of the local minimum point of the graph of f is 3.7 correct to 2 significant figures. [3]

d. Find the range of f. [3]

e. Points A and B lie on the graph of f. The *x*-coordinates of A and B are 1 and 7 respectively. [1]

Write down the *y*-coordinate of B.

f. Points A and B lie on the graph of f . The *x*-coordinates of A and B are 1 and 7 respectively. [2]

Find the gradient of the straight line passing through A and B.

g. M is the midpoint of the line segment AB. [2]

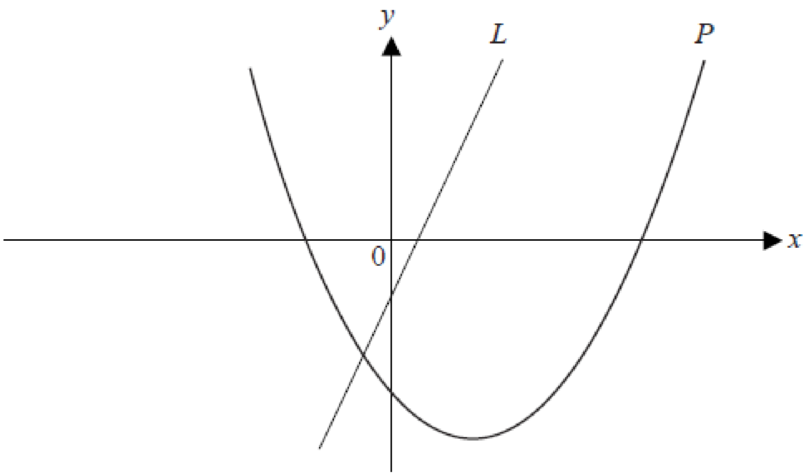
Write down the coordinates of M.

h. *L* is the tangent to the graph of the function $y = f(x)$, at the point on the graph with the same *x*-coordinate as M. [2]

Find the gradient of *L*.

i. Find the equation of *L*. Give your answer in the form $y = mx + c$. [3]

The diagram below shows the graph of a line L passing through (1, 1) and (2 , 3) and the graph P of the function $f(x) = x^2 - 3x - 4$



a. Find the gradient of the line L. [2]

b. Differentiate $f(x)$. [2]

c. Find the coordinates of the point where the tangent to P is parallel to the line L. [3]

d. Find the coordinates of the point where the tangent to P is perpendicular to the line L. [4]

e. Find [3]

(i) the gradient of the tangent to P at the point with coordinates (2, − 6).

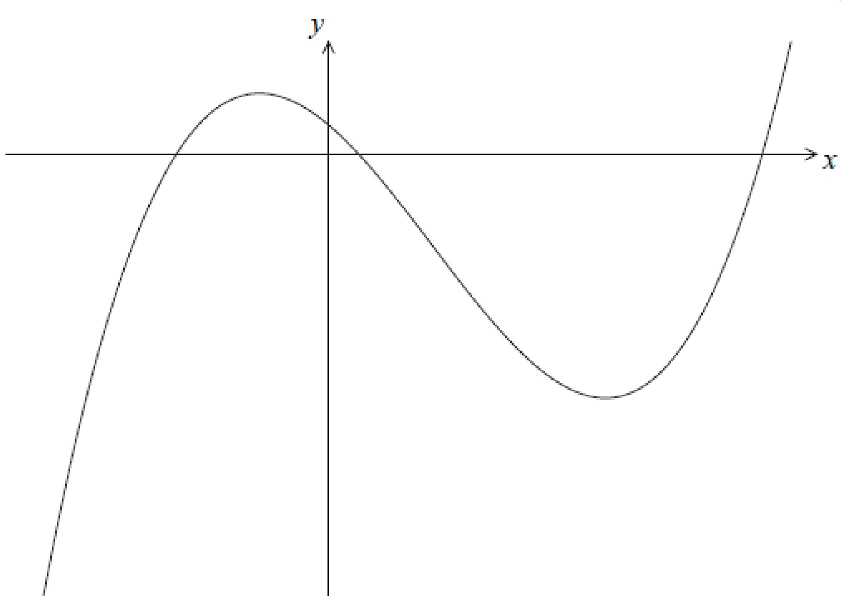
(ii) the equation of the tangent to P at this point.

f. State the equation of the axis of symmetry of P. [1]

g. Find the coordinates of the vertex of P and state the gradient of the curve at this point. [3]

The diagram shows a sketch of the function $f(x) = 4x^3 - 9x^2 - 12x + 3$.

diagram not to scale



a. Write down the values of x where the graph of $f(x)$ intersects the x-axis. [3]

b. Write down $f'(x)$. [3]

c. Find the value of the local maximum of $y = f(x)$. [4]

d. Let P be the point where the graph of $f(x)$ intersects the y axis. [1]

Write down the coordinates of P.

e. Let P be the point where the graph of $f(x)$ intersects the y axis. [2]

Find the gradient of the curve at P.

f. The line, L, is the tangent to the graph of $f(x)$ at P. [2]

Find the equation of L in the form $y = mx + c$.

g. There is a second point, Q, on the curve at which the tangent to $f(x)$ is parallel to L. [1]

Write down the gradient of the tangent at Q.

h. There is a second point, Q, on the curve at which the tangent to $f(x)$ is parallel to L. [3]

Calculate the x-coordinate of Q.

Consider the function $f(x) = x^3 – 3\text{x}– 24x + 30$.

a. Write down $f(0)$. [1]

b. Find $f'(x)$. [3]

c. Find the gradient of the graph of $f(x)$ at the point where $x = 1$. [2]

d. (i) Use $f'(x)$ to find the x-coordinate of M and of N. [5]

(ii) Hence or otherwise write down the coordinates of M and of N.

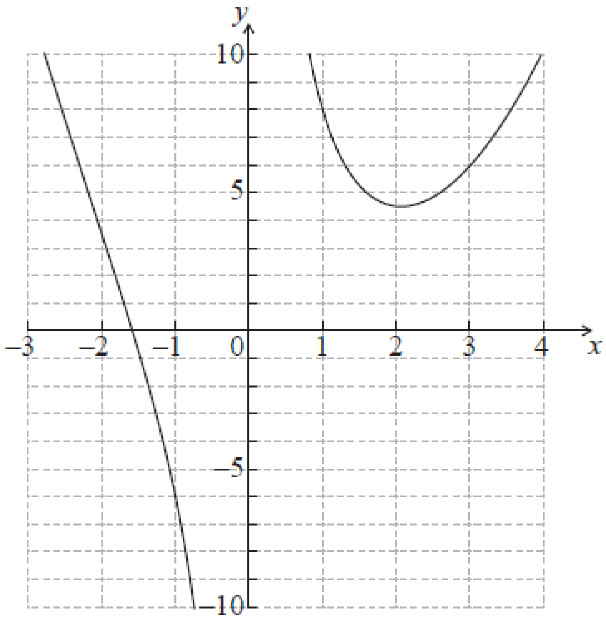
e. Sketch the graph of $f(x)$ for $-5 \leqslant x \leqslant 7$ and $-60 \leqslant y \leqslant 60$. Mark clearly M and N on your graph. [4]

f. Lines L_1 and L_2 are parallel, and they are tangents to the graph of $f(x)$ at points A and B respectively. L_1 has equation $y = 21x + 111$. [6]

(i) Find the x-coordinate of A and of B.

(ii) Find the y-coordinate of B.

The diagram shows part of the graph of $f(x) = x^2 - 2x + \frac{9}{x}$, where $x \neq 0$.



a. Write down [5]

(i) the equation of the vertical asymptote to the graph of $y = f(x)$;

(ii) the solution to the equation $f(x) = 0$;

(iii) the coordinates of the local minimum point.

b. Find $f'(x)$. [4]

c. Show that $f'(x)$ can be written as $f'(x) = \frac{2x^3-2x^2-9}{x^2}$. [2]

d. Find the gradient of the tangent to $y = f(x)$ at the point $\mathrm{A}(1, 8)$. [2]

e. The line, L, passes through the point A and is perpendicular to the tangent at A. [1]

Write down the gradient of L .

f. The line, L , passes through the point A and is perpendicular to the tangent at A. [3]

Find the equation of L . Give your answer in the form $y = mx + c$.

g. The line, L , passes through the point A and is perpendicular to the tangent at A. [2]

L also intersects the graph of $y = f(x)$ at points B and C . Write down the **x-coordinate** of B and of C .

Consider the function $g(x) = bx - 3 + \frac{1}{x^2}$, $x \neq 0$.

a. Write down the equation of the vertical asymptote of the graph of $y = g(x)$. [2]

b. Write down $g'(x)$. [3]

c. The line T is the tangent to the graph of $y = g(x)$ at the point where $x = 1$. The gradient of T is 3. [2]

Show that $b = 5$.

d. The line T is the tangent to the graph of $y = g(x)$ at the point where $x = 1$. The gradient of T is 3. [3]

Find the equation of T.

e. Using your graphic display calculator find the coordinates of the point where the graph of $y = g(x)$ intersects the x-axis. [2]

f. (i) Sketch the graph of $y = g(x)$ for $-2 \le x \le 5$ and $-15 \le y \le 25$, indicating clearly your answer to part (e). [6]

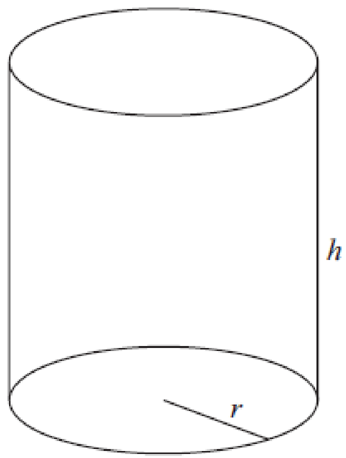
(ii) Draw the line T on your sketch.

g. Using your graphic display calculator find the coordinates of the local minimum point of $y = g(x)$. [2]

h. Write down the interval for which $g(x)$ is increasing in the domain $0 < x < 5$. [2]

Nadia designs a wastepaper bin made in the shape of an **open** cylinder with a volume of 8000 cm^3.

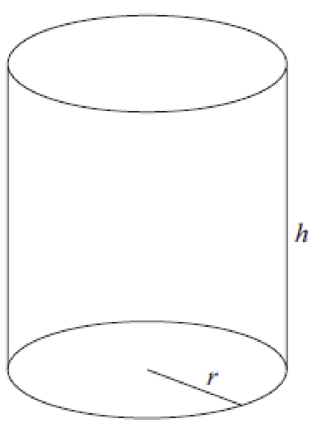
diagram not to scale



Nadia decides to make the radius, r , of the bin 5 cm.

Merryn also designs a cylindrical wastepaper bin with a volume of 8000 cm^3. She decides to fix the radius of its base so that the **total external surface area** of the bin is minimized.

diagram not to scale



Let the radius of the base of Merryn's wastepaper bin be r , and let its height be h .

a. Calculate [7]

(i) the area of the base of the wastepaper bin;

(ii) the height, h , of Nadia's wastepaper bin;

(iii) the total **external** surface area of the wastepaper bin.

b. State whether Nadia's design is practical. Give a reason. [2]

c. Write down an equation in h and r , using the given volume of the bin. [1]

d. Show that the total external surface area, A , of the bin is $A = \pi r^2 + \frac{16000}{r}$. [2]

e. Write down $\frac{\mathrm{d}A}{\mathrm{d}r}$. [3]

f. (i) Find the value of r that minimizes the total external surface area of the wastepaper bin. [5]

(ii) Calculate the value of h corresponding to this value of r .

g. Determine whether Merryn's design is an improvement upon Nadia's. Give a reason. [2]

Consider the function $f(x) = 0.5x^2 - \frac{8}{x},\ x \neq 0.$

a. Find $f(-2)$. [2]

b. Find $f'(x)$. [3]

c. Find the gradient of the graph of f at $x = -2$. [2]

d. Let T be the tangent to the graph of f at $x = -2$. [2]

Write down the equation of T.

e. Let T be the tangent to the graph of f at $x = -2$. [4]

Sketch the graph of f for $-5 \leqslant x \leqslant 5$ and $-20 \leqslant y \leqslant 20$.

f. Let T be the tangent to the graph of f at $x = -2$. [2]

Draw T on your sketch.

g. The tangent, T, intersects the graph of f at a second point, P. [2]

Use your graphic display calculator to find the coordinates of P.

Consider the function $f : x \mapsto \frac{kx}{2^x}$.

The cost per person, in euros, when x people are invited to a party can be determined by the function

$$C(x) = x + \frac{100}{x}$$

i.a. Given that $f(1) = 2$, show that $k = 4$. [2]

i.b. Write down the values of q and r for the following table. [2]

x	-1	0	1	2	4	8
$f(x)$	-8	0	2	q	1	r

i.c. As x increases from -1, the graph of $y = f(x)$ reaches a maximum value and then decreases, behaving asymptotically. [4]

Draw the graph of $y = f(x)$ for $-1 \leqslant x \leqslant 8$. Use a scale of 1 cm to represent 1 unit on both axes. The position of the maximum, \({\text{M}}\), the y-intercept and the asymptotic behaviour should be clearly shown.

i.d. Using your graphic display calculator, find the coordinates of M, the maximum point on the graph of $y = f(x)$. [2]

i.e. Write down the equation of the horizontal asymptote to the graph of $y = f(x)$. [2]

i.f. (i) Draw and label the line $y = 1$ on your graph. [4]

(ii) The equation $f(x) = 1$ has two solutions. One of the solutions is $x = 4$. Use your **graph** to find the other solution.

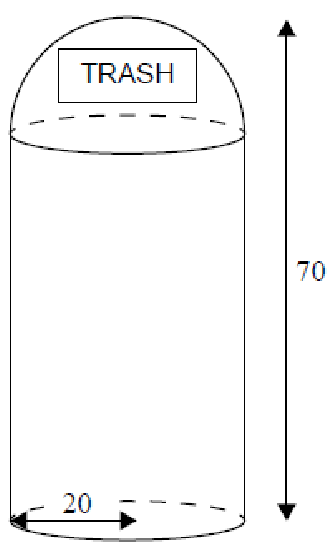
ii.a. Find $C'(x)$. [3]

ii.b. Show that the cost per person is a minimum when 10 people are invited to the party. [2]

ii.c. Calculate the minimum cost per person. [2]

A manufacturer makes trash cans in the form of a cylinder with a hemispherical top. The trash can has a height of 70 cm. The base radius of both the cylinder and the hemispherical top is 20 cm.

diagram not to scale

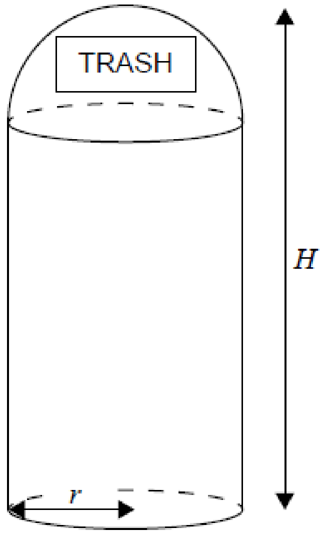


A designer is asked to produce a new trash can.

The new trash can will also be in the form of a cylinder with a hemispherical top.

This trash can will have a height of H cm and a base radius of r cm.

diagram not to scale



There is a design constraint such that $H + 2r = 110$ cm.

The designer has to maximize the volume of the trash can.

a. Write down the height of the cylinder. [1]

b. Find the total volume of the trash can. [4]

c. Find the height of the **cylinder**, h , of the new trash can, in terms of r. [2]

d. Show that the volume, V cm^3 , of the new trash can is given by [3]

$V = 110\pi r^3$.

e. Using your graphic display calculator, find the value of r which maximizes the value of V. [2]

f. The designer claims that the new trash can has a capacity that is at least 40% greater than the capacity of the original trash can. [4]

State whether the designer's claim is correct. Justify your answer.

Consider the curve $y = 2x^3 - 9x^2 + 12x + 2$, for $-1 < x < 3$

a. Sketch the curve for $-1 < x < 3$ and $-2 < y < 12$. [4]

b. A teacher asks her students to make some observations about the curve. [1]

Three students responded.
Nadia said *"The x-intercept of the curve is between –1 and zero".*
Rick said *"The curve is decreasing when x < 1 ".*
Paula said *"The gradient of the curve is less than zero between x = 1 and x = 2 ".*

State the name of the student who made an **incorrect** observation.

c. Find the value of y when $x = 1$. [2]

d. Find $\frac{dy}{dx}$. [3]

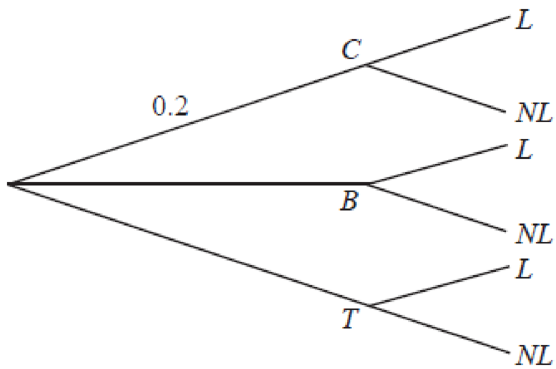
e. Show that the stationary points of the curve are at $x = 1$ and $x = 2$. [2]

f. Given that $y = 2x^3 - 9x^2 + 12x + 2 = k$ has **three** solutions, find the possible values of k. [3]

When Geraldine travels to work she can travel either by car (C), bus (B) or train (T). She travels by car on one day in five. She uses the bus 50 % of the time. The probabilities of her being late (L) when travelling by car, bus or train are 0.05, 0.12 and 0.08 respectively.

*It is **not** necessary to use graph paper for this question.*

i.a. Copy the tree diagram below and fill in all the probabilities, where *NL* represents not late, to represent this information. [5]



i.b. Find the probability that Geraldine travels by bus and is late. [1]

i.c. Find the probability that Geraldine is late. [3]

i.d. Find the probability that Geraldine travelled by train, given that she is late. [3]

ii.a. Sketch the curve of the function $f(x) = x^3 - 2x^2 + x - 3$ for values of x from −2 to 4, giving the intercepts with both axes. [3]

ii.b. On the same diagram, sketch the line $y = 7 - 2x$ and find the coordinates of the point of intersection of the line with the curve. [3]

ii.c. Find the value of the gradient of the curve where $x = 1.7$. [2]

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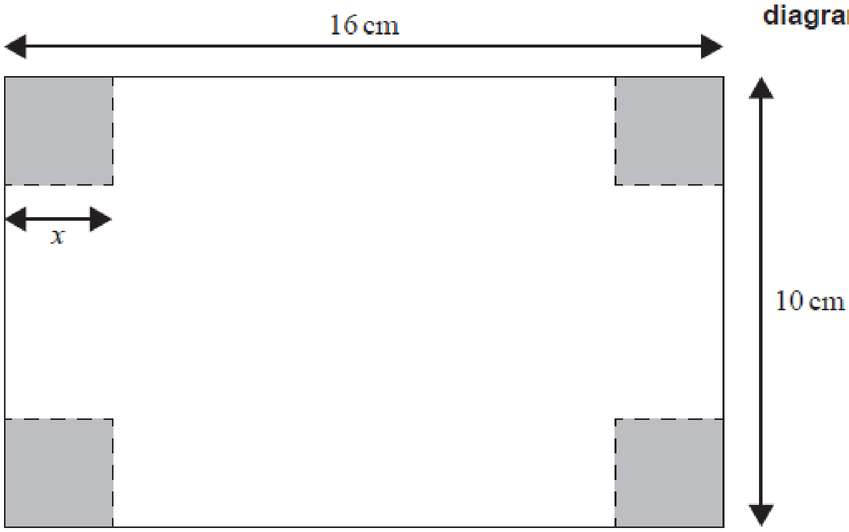
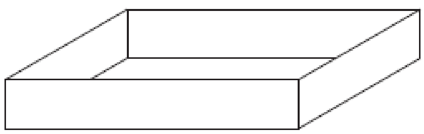
a. Hugo is given a rectangular piece of thin cardboard, 16 cm by 10 cm. He decides to design a tray with it. [2]

He removes from each corner the shaded squares of side x cm, as shown in the following diagram.



The remainder of the cardboard is folded up to form the tray as shown in the following diagram.



Write down, **in terms of** x , the length and the width of the tray.

b. (i) State whether x can have a value of 5. Give a reason for your answer. [4]

(ii) Write down the interval for the possible values of x .

c. Show that the volume, $V\ \mathrm{cm}^3$, of this tray is given by [2]

$$V = 4x^3 - 52x^2 + 160x.$$

d. Find $\frac{dV}{dx}$. [3]

e. **Using your answer from part (d)**, find the value of x that maximizes the volume of the tray. [2]

f. Calculate the maximum volume of the tray. [2]

g. Sketch the graph of $V = 4x^3 - 51x^2 + 160x$, for the possible values of x found in part (b)(ii), and $0 \leqslant V \leqslant 200$. Clearly label the maximum point. [4]

---

a. A function, f , is given by [2]

$$f(x) = 4 \times 2^{-x} + 1.5x - 5.$$

Calculate $f(0)$

b. Use your graphic display calculator to solve $f(x) = 0$. [2]

c. Sketch the graph of $y = f(x)$ for $-2 \leqslant x \leqslant 6$ and $-4 \leqslant y \leqslant 10$, showing the x and y intercepts. Use a scale of $2\ \mathrm{cm}$ to represent 2 units on both the horizontal axis, x , and the vertical axis, y . [4]

d. The function f is the derivative of a function g . It is known that $g(1) = 3$. [4]

i) Calculate $g'(1)$.

ii) Find the equation of the tangent to the graph of $y = g(x)$ at $x = 1$. Give your answer in the form $y = mx + c$.

Consider the function $f(x) = \frac{48}{x} + kx^2 - 58$, where $x > 0$ and k is a constant.

The graph of the function passes through the point with coordinates (4 , 2).

P is the minimum point of the graph of $f(x)$.

a. Find the value of k. [2]

b. Using your value of k , find $f'(x)$. [3]

c. **Use your answer** to part (b) to show that the minimum value of $f(x)$ is −22 . [3]

d. Write down the **two** values of x which satisfy $f(x) = 0$. [2]

e. Sketch the graph of $y = f(x)$ for $0 < x \leq 6$ and $-30 \leq y \leq 60$. [4]

Clearly indicate the minimum point P and the x-intercepts on your graph.

---

a. A distress flare is fired into the air from a ship at sea. The height, h , in metres, of the flare above sea level is modelled by the quadratic function [1]

$$h(t) = -0.2t^2 + 16t + 12\,,\ t \geqslant 0\,,$$

where t is the time, in seconds, and $t = 0$ at the moment the flare was fired.

Write down the height from which the flare was fired.

b. Find the height of the flare 15 seconds after it was fired. [2]

c. The flare fell into the sea k seconds after it was fired. [2]

Find the value of k .

d. Find $h'(t)$. [2]

e. i) Show that the flare reached its maximum height 40 seconds after being fired. [3]

ii) Calculate the maximum height reached by the flare.

f. The nearest coastguard can see the flare when its height is more than 40 metres above sea level. [3]

Determine the total length of time the flare can be seen by the coastguard.

Consider the function $f(x) = 3x + \frac{12}{x^2},\ x \neq 0$.

a. Differentiate $f(x)$ with respect to x. [3]

b. Calculate $f'(x)$ when $x = 1$. [2]

c. Use your answer to part (b) to decide whether the function, f, is increasing or decreasing at $x = 1$. Justify your answer. [2]

d. Solve the equation $f'(x) = 0$. [3]

e, i. The graph of *f* has a local minimum at point P. Let *T* be the tangent to the graph of *f* at P. [2]

Write down the coordinates of P.

e, ii. The graph of *f* has a local minimum at point P. Let *T* be the tangent to the graph of *f* at P. [1]

Write down the gradient of *T*.

e, iii. The graph of *f* has a local minimum at point P. Let *T* be the tangent to the graph of *f* at P. [2]

Write down the equation of *T*.

f. Sketch the graph of the function *f*, for –3 ≤ *x* ≤ 6 and –7 ≤ *y* ≤ 15. Indicate clearly the point P and any intercepts of the curve with the axes. [4]

g, i. On your graph draw and label the tangent *T*. [2]

g, ii. *T* intersects the graph of *f* at a second point. Write down the *x*-coordinate of this point of intersection. [1]

Consider the function $f(x) = -\frac{1}{3}x^3 + \frac{5}{3}x^2 - x - 3$.

a. Sketch the graph of *y* = *f* (*x*) for –3 ≤ *x* ≤ 6 and –10 ≤ *y* ≤ 10 showing clearly the axes intercepts and local maximum and minimum points. Use a scale of 2 cm to represent 1 unit on the *x*-axis, and a scale of 1 cm to represent 1 unit on the *y*-axis. [4]

b. Find the value of *f* (–1). [2]

c. Write down the coordinates of the *y*-intercept of the graph of *f* (*x*). [1]

d. Find *f* '(*x*). [3]

e. Show that $f'(-1) = -\frac{16}{3}$. [1]

f. Explain what *f* '(–1) represents. [2]

g. Find the equation of the tangent to the graph of *f* (*x*) at the point where *x* is –1. [2]

h. Sketch the tangent to the graph of *f* (*x*) at *x* = –1 on your diagram for (a). [2]

i. P and Q are points on the curve such that the tangents to the curve at these points are horizontal. The x-coordinate of P is a, and the x-coordinate of Q is b, $b > a$. [2]

Write down the value of

(i) a ;

(ii) b .

j. P and Q are points on the curve such that the tangents to the curve at these points are horizontal. The x-coordinate of P is a, and the x-coordinate of Q is b, $b > a$. [1]

Describe the behaviour of $f(x)$ for $a < x < b$.